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APPLICATION OF QUEUEING THEORY
TO
THE REVIEW AND VALIDATION OF
FOREIGN MILITARY SALES (FMS) CASES

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TECHNICAL REPORT

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ABSTRACT

The application of queueing, or waiting-line, theory to the review
and validation of Foreign Military Sales (FMS) cases by the Cost
Analysis office of an Army Materiel Development and Readiness Command
major subordinate command is presented in the report. Queueing
analysis is applied to determine the behavior characteristics of
the processing system so that management can take appropriate
action to reduce the total time in the system. The primary charac-
teristics analyzed are the expected length of the queue, the expected
number of cases in the system, the expected waiting time, and the
expected turnaround time. Queueing theory is described, and the
application to FMS cases presented, to also serve as an aid in the
solution of similar types of waiting-line problems.

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A. Purpose

This paper is presented to describe the use of queueing, or waiting line, theory in the analysis of turnaround time in the review and validation of Foreign Military Sales (FMS) cases within the Data Analysis and Validation Branch (DRSAR-CPE-D) of the HQ, ARMCOM Cost Analysis Division. The method of analysis described is applicable to a variety of similar work situations.

B. Queueing Theory

An important class of management problem includes those that can be characterized as "arrival and departure" problems. These types of problem occur whenever randomly arriving customers are required to wait for some common type of service. The term customer can denote people, paperwork, machines, or any other kind of discrete arrival, whereas the term service applies to any kind of work performed on or for the customer. Examples are customers arriving at a doctor's office, machines waiting to be repaired, and paperwork waiting to be processed. Given that the service facility or facilities are adequate to meet the demand for service, and are not excessive relative to the demand, a condition exists in which both customers and service facilities occasionally encounter temporary waiting owing to variations in service times and/or arrival rates. When customers are required to wait, they form a waiting line, or queue; hence, the analytical techniques used to solve waiting-line problems are embodied in what is commonly referred to as queueing theory.

Patterns of service and/or arrival rates are, under certain assumptions, susceptible to approximation by mathematically defined frequency distributions. On this basis, mathematical models have been formulated which can be used to determine, on a probabilistic basis, the behavior characteristics of arrival-departure systems that meet the required conditions and assumptions. Repeated execution of the models, varying the number of service facilities or stations, and, if feasible, the mean (average) service time, provides the behavior pattern of the system being analyzed. These data can be used to indicate those actions management can take to minimize the total time customers spend in the system, and/or the total cost of waiting.

The simplest kind of waiting line system are those in which customers arrive at a single service facility, occasionally have to wait for service, and are serviced according to some priority rule such as "first-come, first-served". This type of system can be called a single-station, single-stage system. More complex kinds of system can exist, such as single-station, multiple-stage; multiple-station, single-stage; and multiple-station, multiple-stage, or series parallel.

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The models formulated for queueing analysis vary depending on the type of system, although each has its basis in the type of arrival-rate and service-rate frequency distributions assumed for waiting lines. The type of model and the input parameters which are applicable to the analysis of FMS cases are described in paragraph C3.

C. Application to FMS Cases

1. Purpose of the Analysis

Queueing theory has been applied to the review and validation of FMS cases by DRSAR-CPE-D to provide management with the behavior characteristics of the review and validation process so that action could be taken to reduce the total expected time an individual case spends in the system from receipt to completion, i.e., the expected turnaround time. The relevant characteristics are the average time in the system and the number of reviewers required to reduce or minimize the average time.

2. Type of System

Cases arrive randomly from the HQ, ARMCOM International Logistics Directorate (DRSAR-IL). Following receipt, they are logged in and assigned to one of several analysts. A case may or may not have to wait, depending on when it arrives relative to the backlog and cases in process in the Branch at the time. In general, FMS cases are reviewed and validated to ensure compliance with pricing regulations and policy, verify the correct application of inflation indices, guarantee full cost recovery, assure fair prices to both parties, ensure proper calculation of unfunded costs, and to reduce the number of amendments necessary because of price fluctuations. The same basic process is followed in the review and validation of each case, although they differ in scope and hence vary randomly in the processing time required. In actual practice, a case may be started and later delayed pending receipt of essential information; hence, an analyst may have more than one case under review at any one time. Following the review and validation process, formulation of the analyst's initial recommendation, and Branch Chief review and recommendation, decision for formal action is made by the Division Chief, Comptroller, or Deputy Comptroller, depending on the dollar value of the case and whether concurrence or non-concurrence is indicated. The case is then released for return to DRSAR-IL.

The purpose of the analysis is to determine the characteristics of the analyst's review and validation process, not the approval process. This restricts the definition of the system to the latter, and simplifies the analysis, since a separate approval/decision step creates a multiple-stage system. As a result, the output of the analysis can only guide management actions to reduce turnaround time through completion by the analyst. The contribution to total elapsed

time through release to DRSAR-IL created by the approval stage is subject to separate analysis. The review and validation system thus described is a multiple-station, single-stage system, as shown in Figure 1. The elapsed time from receipt by the Branch to the start of processing, and from receipt by the Branch to completion by the analyst, constitutes the waiting time and turnaround time, respectively. The service time consists of the actual time the analyst spends processing an individual case.

3.   Applicable Model

In the model described in this paper for a multiple-station, single-stage system, each station, k, is an individual analyst where $k > 1$, and each case is processed by a single station or analyst. Queueing models for this type of system involve consideration of the mean arrival rate (λ) and the mean service or processing rate (μ) to determine the expected waiting time which, when added to the mean service time, provides the total expected time in the system. The model assumes:

a.   Arrivals are random and are Poisson distributed with a mean number of arrivals, λ, per unit of time.

b.   Service times are constant or exponentially distributed with a mean service time, $1/\mu$; i.e., the reciprocal of the mean service rate.

c.   The service priority is first come, first served.

d.   The number of service stations (analysts) is known, and all stations have the same service capacity.

e.   The mean service time (processing speed) is unaffected by the length of the queue; i.e., service times are statistically independent.

f.   There is an infinite source of inputs.

Given that these assumptions are true, the queueing models will provide, on a probabilistic basis, measures of the system characteristics that are suitable for management analysis and decision.

The utilization parameter of the system is defined as the proportion of time that all k analysts are being utilized, and assumes that $k\mu > \lambda$. Note that if this assumption is incorrect that the k analysts will be theoretically busy more than 100 percent of the time, and an infinite queue will build up. The parameter, denoted by the Greek letter rho, is expressed as

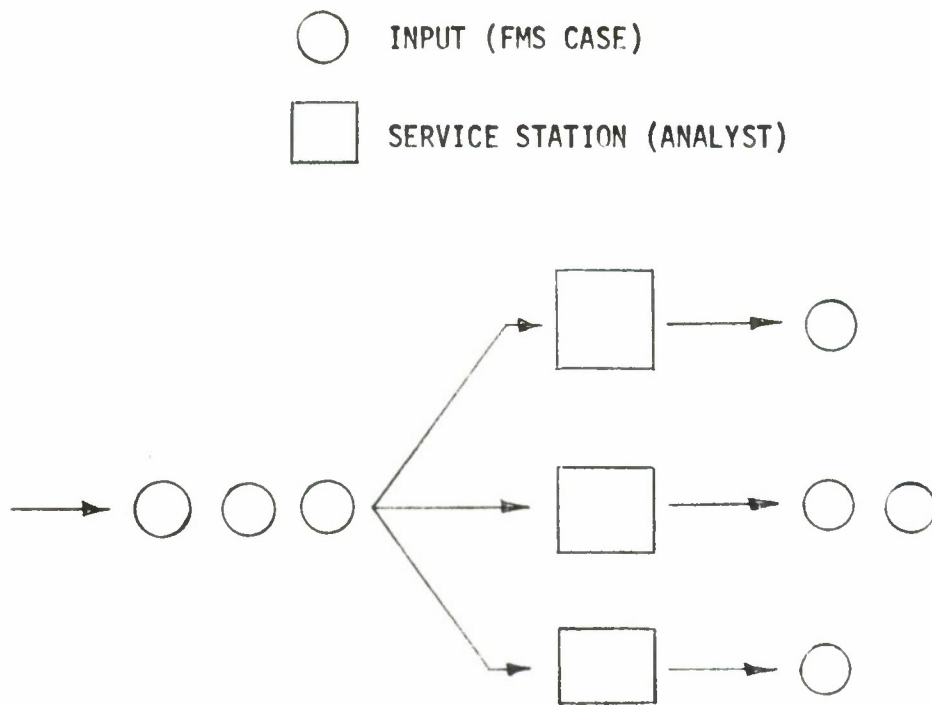$$\rho_k = \frac{\lambda}{k\mu}$$

○ INPUT (FMS CASE)

□ SERVICE STATION (ANALYST)

FIGURE 1. FMS CASE PROCESSING SYSTEM: Multiple-Station, Single-Stage

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The probability that all k analysts are idle, i.e., the probability that there is no case in the system, is defined as

$$P_0 = \cfrac{1}{\left[\displaystyle\sum_{i=0}^{k-1} \frac{1}{i!}\left(\frac{\lambda}{\mu}\right)^i \right] + \left[\frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \left(\frac{k\mu}{k\mu - \lambda}\right) \right]}$$

The probability that a randomly arriving case has to wait for processing, i.e., the probability that there are k or more cases in the system, is

$$P_{n>k} = P_0 \left[\frac{\mu \, (\lambda/\mu)^k}{(k-1)! \, (k\mu - \lambda)} \right]$$

The mean (expected) length of the queue, i.e., the number of cases waiting for processing is

$$E(m) = P_0 \left[\frac{\lambda \mu (\lambda/\mu)^k}{(k-1)! \, (k\mu - \lambda)^2} \right]$$

The mean (expected) number of cases in the system is

$$E(n) = E(m) + \frac{\lambda}{\mu}$$

The average (expected) waiting time of an arriving case in the queue, expressed in the same <u>time</u> units as λ and μ, is

$$E(w) = P_0 \left[\frac{\mu \, (\lambda/\mu)^k}{(k-1)! \, (k\mu - \lambda)^2} \right] = \frac{E(m)}{\lambda}$$

The average (expected) time a case spends in the system, i.e., the expected turnaround time, expressed in the same <u>time</u> units as λ and μ, becomes

$$E(t) = E(w) + \frac{1}{\mu}$$

4. Data Collection and Analysis

To provide inputs to the models, the arrival and processing distributions were analyzed as follows:

a. Data for numbers of arrivals per day were collected for the period 2 Jan through 14 Apr 76, and are shown in Table 1 by month. The distribution of observed frequencies of arrival determined from the data is also shown in Table 2. The cumulative mean arrival rate

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TABLE 1. FMS CASE ARRIVALS (2 JAN - 14 APR 76)

No. Cases Per Day	Number of Days				
	January	February	March	April	TOTAL
0			1		1
1	2		1	1	4
2	2	3	2		7
3	5	3	1	1	10
4	8	5	1	1	15
5	4	1	4	1	10
6		3	5	2	10
7		2	4		6
8		2	4	1	7
9				1	1
10				1	1
11					
12					
13					
14				1	1
TOTAL	21	19	23	10	73

SOURCE: DRSAR-CPE-D daily logs, 2 Jan - 14 Apr 76.

TABLE 2. FREQUENCY DISTRIBUTION - FMS CASES (2 JAN - 14 APR 76)

No. of Cases Arriving Per Day (x)	Observed Frequency (f_o)	$x \cdot f_o$	Expected Frequency (f_e) ($\lambda = 4.80$)
0	1	0	0.60
1	4	4	2.88
2	7	14	6.92
3	10	30	11.07
4	15	60	13.30
5	10	50	12.75
6	10	60	10.21
7	6	42	7.00
8	7	56	4.20
9	1	9	2.24
10	1	10	1.07
11	0	0	0.47
12	0	0	0.19
13	0	0	0.07
14	1	14	0.02
15 or more	0	0	0.01
TOTAL	73	349	73.00

$$\text{Mean Arrival Rate} = \frac{\sum x f_o}{\sum f_o} = \frac{349}{73} = 4.78 \text{ cases/day}$$

$$f_e = \sum f_o \left[P(r=x \mid \lambda = 4.80) \right]$$

was then calculated as shown. Since the mean arrival rate parameter, λ, and the total frequency, $\sum f_o$, are sufficient to completely describe a Poisson distribution, expected frequencies, f_e, for a Poisson distribution with $\lambda = 4.80$ were calculated using a table of Poisson probabilities of the type found in most basic statistics texts, and are also listed in Table 2. The value of $\lambda = 4.80$ was the closest table value to the calculated value of 4.78. Observed and expected frequencies were plotted, Figure 2, showing that arrivals over the period plotted are closely approximated by a Poisson distribution with $\lambda = 4.80$. A statistical test of significance, called the chi-square test, can be used to determine the goodness of fit of the distribution of observed frequencies to the Poisson distribution, and is described in Appendix A for the analysis of FMS case arrivals.

     b.   Reliable observed or estimated processing time per case data were not available. It is known that processing times are not uniformly distributed; therefore, an exponential distribution was assumed. Since arrivals fit the model assumption, this assumption of service time distribution was considered reasonable. The single parameter required to describe an exponential distribution is the mean processing time, $1/\mu$. An estimate of μ was obtained by dividing the total hours expended on FMS cases during the period January 1976 through March 1976 by the total number of cases reviewed during the same period. These data are shown in Table 3. Estimates of μ are also shown by month, and cumulatively through the period analyzed (see paragraph C4c). Conversion to a mean processing rate, in cases per day per person, was necessary to provide input to the model to match the mean arrival rate parameter in cases per day. The processing times and rates obtained are gross values in that all times expended in both direct and indirect support of case reviews, including resolution of policy, meetings, and administrative activities, is included.

     c.   The arrival and processing data were analyzed to determine the stability of the derived parameters over the period covered by the data. Both monthly and cumulative mean arrival and processing rates were calculated as shown in Table 4 and 3, respectively. These analyses show that, over the periods covered, the mean arrival rate increased while the mean processing time decreased. These trends are plotted in Figure 3. It would be expected that further investigation would indicate the reasons for these trends. For example, training of analysts over the period studied, with attendant learning, might explain the downward trend of mean processing time.

     5.   Solution of the Model

     The model for multiple-station, single-stage waiting line systems has been programmed on the WANG 700 Series programmable calculator located in the Cost Analysis Division. The inputs are k, the number of stations (analysts), in whole integers; λ (lambda), the mean arrival rate, in cases per day; and μ (mu), the mean processing rate per person, in cases per day. When $k\mu < \lambda$, the program will not execute.

FIGURE 2. DISTRIBUTION OF FMS CASE ARRIVALS (2 JAN – 14 APR 76)

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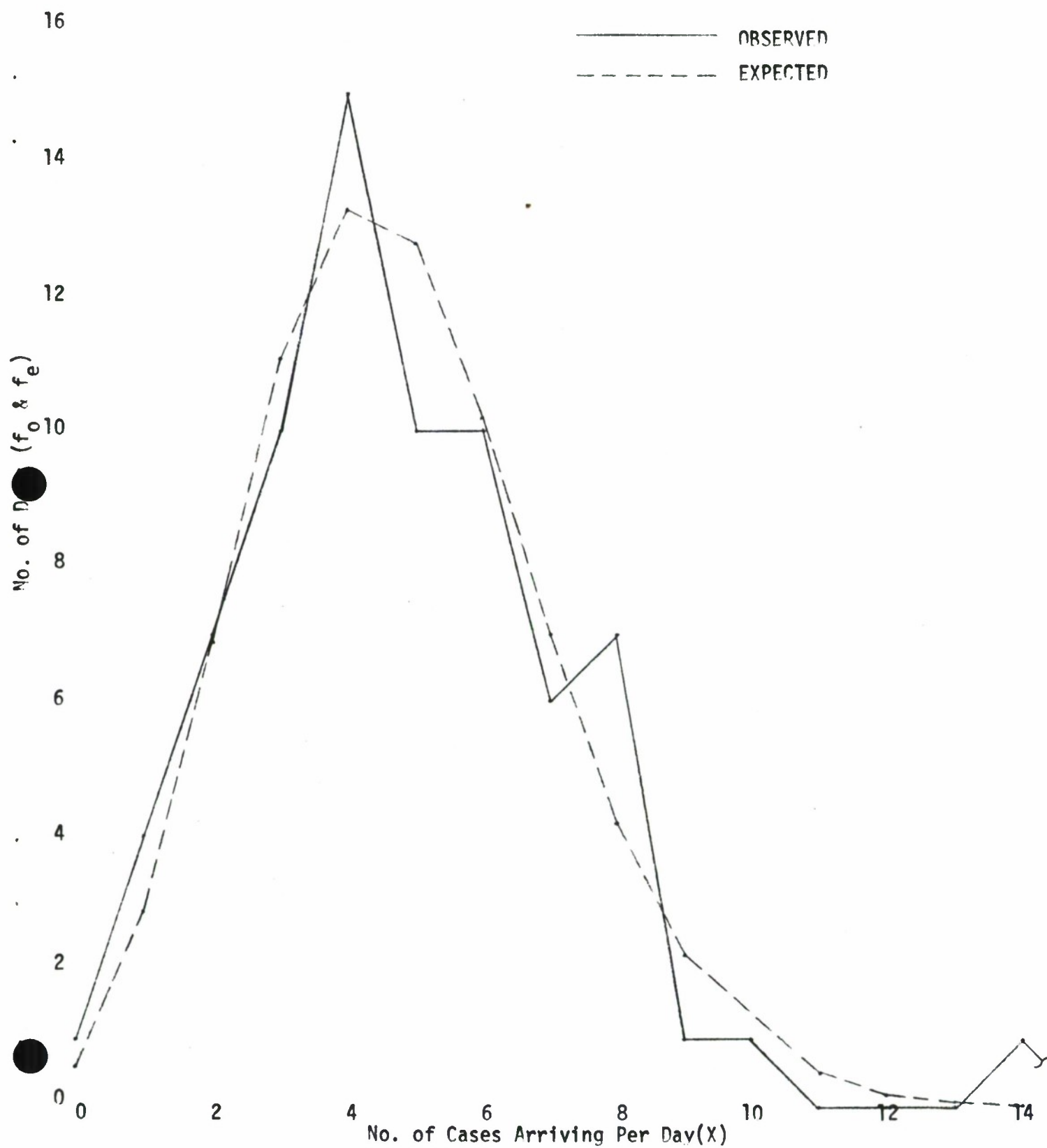
TABLE 3. FMS CASE PROCESSING HOURS (JAN - MAR 76)

	January	February	March	Total
Man-Hours Expended 1/	586	667	626	1,879
No. Cases Reviewed 2/	68	88	114	270

Monthly Mean Processing Times $(1/\mu)$ and Mean Rates (μ)

Jan 76: $\frac{586}{68}$ = 8.618 hours per case (0.928 cases per day per person)

Feb: $\frac{667}{88}$ = 7.580 hours per case (1.055 cases per day per person)

Mar: $\frac{626}{114}$ = 5.491 hours per case (1.457 cases per day per person)

Cumulative Mean Processing Times $(1/\mu)$ and Mean Rates (μ)

Jan 76: See Above

Jan-Feb: $\frac{1253}{156}$ = 8.032 hours per case (0.996 cases per day per person)

Jan-Mar: $\frac{1879}{270}$ = 6.959 hours per case (1.150 cases per day per person)

1/ Source: Labor Tally (Detail), ARMCOM Form 1.

2/ Source: DRSAR-CPE-D periodic reports.

TABLE 4. FMS CASE MONTHLY AND CUMULATIVE

MEAN ARRIVAL RATES (2 JAN - 14 APR 76)

Month(No. Days)		No. of Arrivals	Mean Arrivals Per Day(λ)
Jan 76	(21)	73	3.48
Feb	(19)	88	4.63
Mar	(23)	122	5.30
1-14 Apr	(10)	66	6.60
		349	

Cumulative Mean Arrival Rates (λ)

Jan 76 : $\frac{73}{21}$ = 3.48 cases per day

Jan-Feb : $\frac{161}{40}$ = 4.03 cases per day

Jan-Mar : $\frac{283}{63}$ = 4.49 cases per day

Jan-14 Apr : $\frac{349}{73}$ = 4.78 cases per day

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FIGURE 3.   FMS CASE MEAN ARRIVAL RATE AND PROCESSING TIME

The outputs are the system characteristics defined in paragraph C3.
The program facilitates iterative solution of the model for various
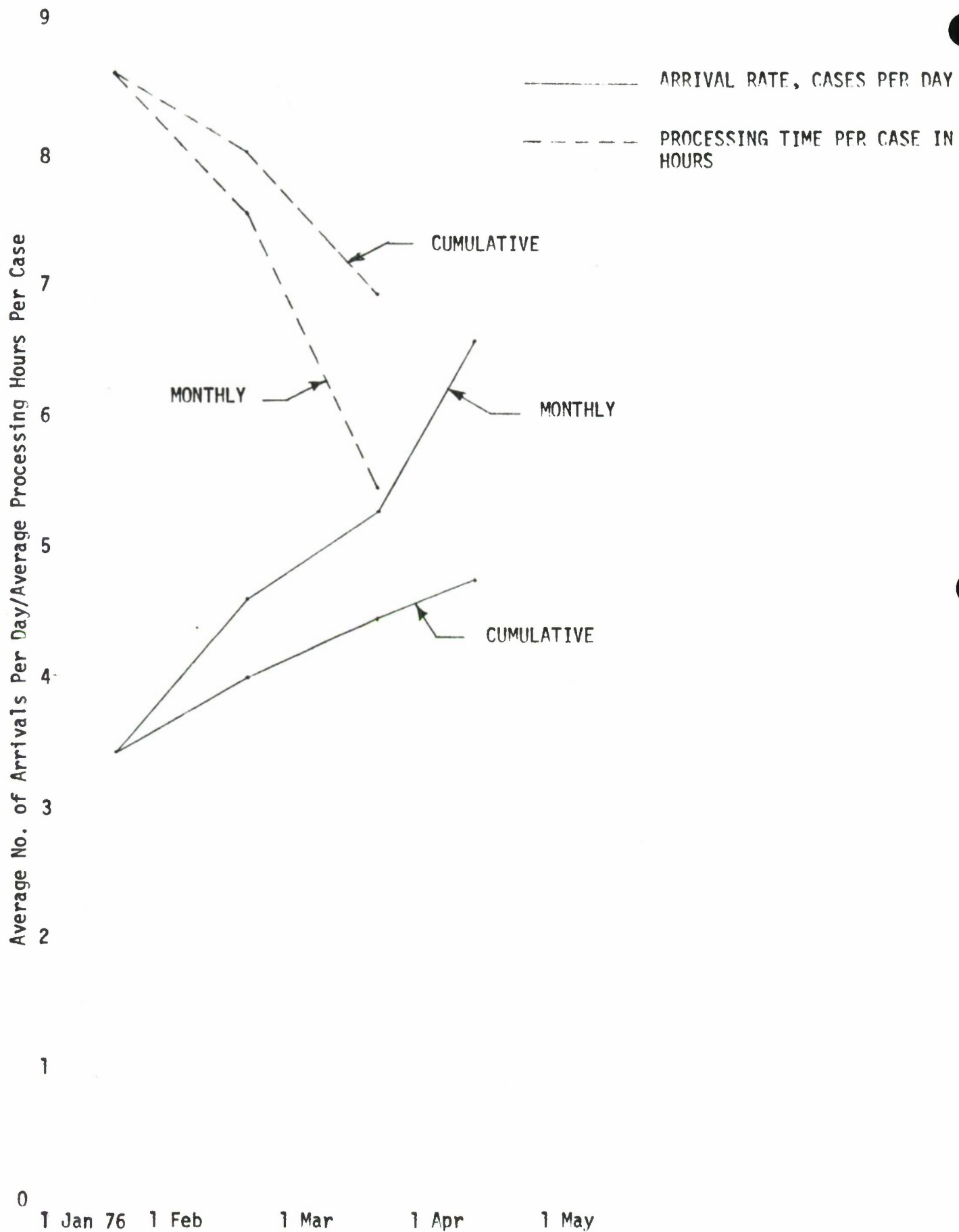values of the inputs, and is expecially useful when trends exist
as discussed in paragraph C4c. In this situation, management must
decide if the system parameters have stabilized or if further analysis
is necessary. As an interim solution, the model has been solved using
values of λ and μ for both the entire period covered by the data and
for the month of March only, and increasing values of k. Printouts
of both solutions are included as Appendix B.

6.   Analysis of the Solution

Since the mean arrival rate is independent of what happens
in the processing system itself, queueing analysis can only indicate
how waiting time, E(w), and total time in the system, E(t), is affected
by changes in the number of analysts, k, and/or in the mean processing
rate, μ. Given a stable value of the latter as well as the arrival
rate, adding resources can only reduce the total expected (turnaround)
time to that of the mean processing time, $1/\mu$. The solutions of
Appendix B show this to occur at k = 10 and k = 11, when the input
parameters are for March and the total period studied, respectively,
reading E(t) to three decimal places. These values are obviously
unrealistic, if only because utilizations are less than 40 percent at
these staffing levels. It is then helpful to management decision-making
to plot expected turnaround time as a function of k. These are shown
in Figure 4. Both plots show graphically that turnaround time drops
rapidly from the lowest value of k at which $k\mu > \lambda$ as analysts are
added. Once management has determined stable values of the input
parameters, this type of analysis is very helpful in determining the
staffing level required to achieve the desired turnaround time, and
the cost penalties of achieving small additional increments of reduced
time.

Solution of the model also shows significant changes in turnaround
time achievable by reduction in the mean processing time, when such
reductions are possible. For example, using the March arrival rate
for FMS cases, if the mean processing rate can be increased from 1.46
cases per day per person to 2.00 cases per day per person, the calculated
turnaround time at k = 4 is reduced by 71 percent.

FIGURE 4. EXPECTED TURNAROUND TIME, IN DAYS, AS A FUNCTION OF THE NUMBER OF ANALYSTS

NOTE: 1. Upper curve based on mean arrival rate, $\lambda = 4.78$, and mean processing rate, $\mu = 1.15$; lower curve based on $\lambda = 5.30$ and $\mu = 1.46$.

2. Lower limit is where k=5 and k=4 for upper and lower curve, respectively, since queue length becomes infinite when $\lambda > k\mu$.

E(t)=Expected Turnaround Time, In Days

2 Jan - 14 Apr 76

Mar 76

k=No. of Analysts

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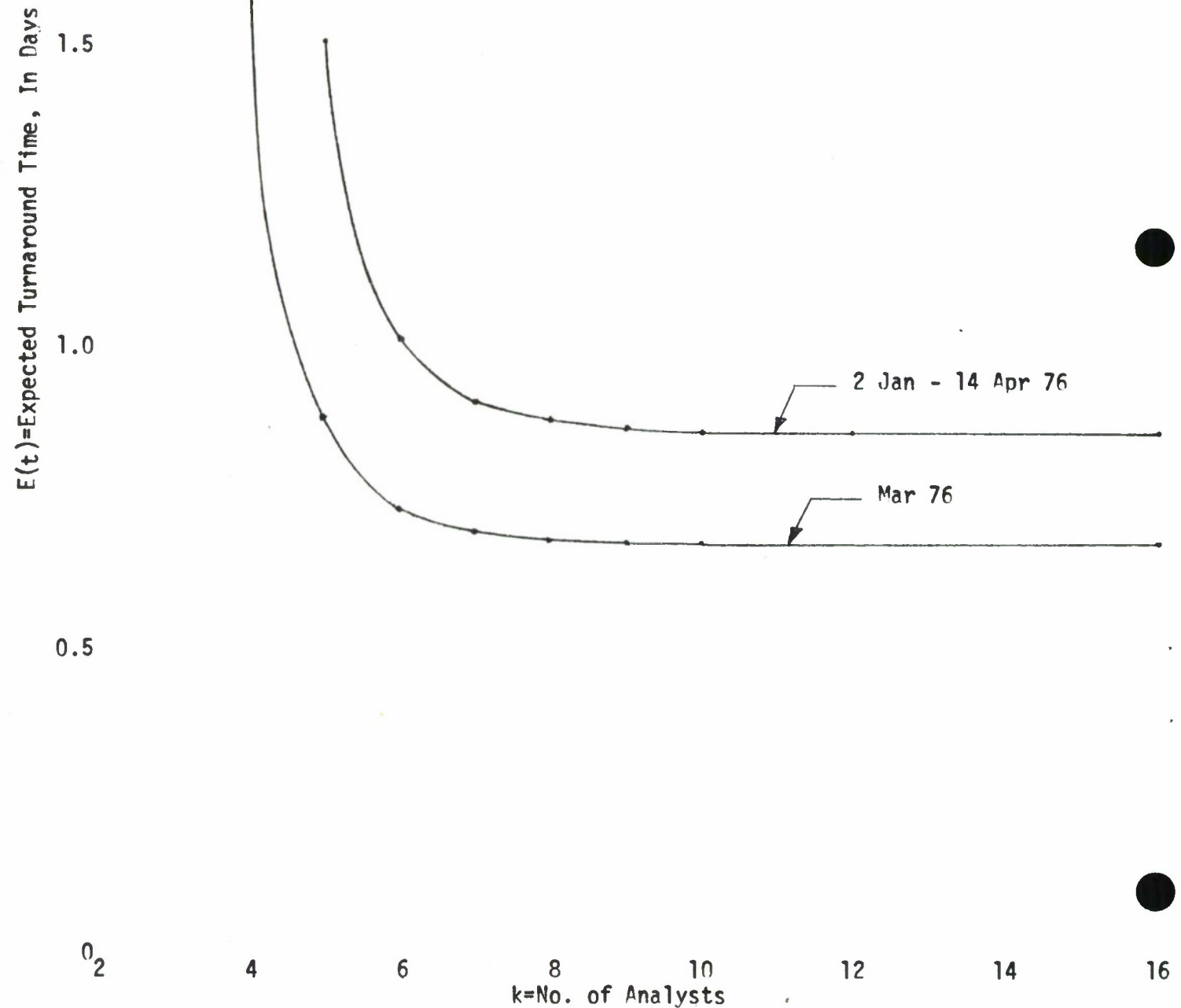
D. <u>References</u>

1. Chou, Ya-lun. <u>Statistical Analysis</u>, 2nd ed. New York: Holt, Rinehart and Winston, 1975.

2. Clark, Charles T., and Lawrence L. Schkade. <u>Statistical Analysis for Administrative Decisions</u>, 2nd ed. Cincinnati: South-Western Publishing Co., 1974.

APPENDIX A

CHI-SQUARE SIGNIFICANCE TEST

CHI-SQUARE TEST OF GOODNESS OF FIT

In testing for goodness of fit, it is necessary to compare the distribution of observed samples with the theoretical distribution that is assumed to be the population from which the sample was drawn. If there is a high degree of conformity, any small differences between the observed and theoretical distributions are assumed to be the result of sampling variation. If there are large differences, the conclusion may be made that the observed sample data was drawn from a population distribution other than that assumed in the test.

In the case of queueing models, arrivals are assumed to be Poisson distributed. For FMS cases, goodness of fit of the observed arrival data to the theoretical distribution was determined graphically by the plot of Figure 2. However, if it is desired to test the goodness of fit statistically, the chi-square test may be used. The chi-square distribution is expressed as

$$\chi^2 = \frac{(f_o - f_e)^2}{f_e}$$

where f_o = an observed frequency, and f_e = a theoretical (expected) frequency. The distribution characteristics are completely defined by the number of degrees of freedom (d.f.), which can be defined as the number of frequency classes, or groupings of data, being compared, minus one d.f. for each restriction placed on the expected distribution. In the case of the Poisson distribution, since it can be completely determined by λ and the total frequency, there will be two restrictions.

Four conditions should be met in order to apply a valid chi-square test. The first two state that the sample observations should be independent of one another and drawn from the population being analyzed, and that the data are usually of some nominal, or moderate, measurement. The remaining two place some quantitative restrictions on the data and resulting frequency classes, in that the sample should contain no fewer than fifty observations and grouped with at least five observations in each frequency class. Statisticians differ on the exact numbers in the latter two conditions. However, fifty or more observations should be possible for most waiting line systems, and grouping of observed data into classes of five or more should normally present no problem. Information obtained by discussion with Mr. Harold Gehle of the US Army Management Engineering Training Agency indicates that the test may be considered valid with as few as three observations in any expected class.

The chi-square test, as with other tests of significance, is set up by establishing a null hypothesis, H_o, that the sample is drawn from the theoretical population distribution, and an alternate hypothesis, H_a, that it is not. The value of chi-square is calculated

from the sample data using the formula on the previous page. A test criterion is then established upon which to base acceptance of either hypothesis. The criterion is based on the value taken from a table of values of chi-square of the type found in most basic statistics texts. The table value is selected on the basis of both the number of degrees of freedom and the degree of risk, called α, that management is willing to take that the test statistic will exceed the table value strictly by chance, thus leading to a rejection of H_o (acceptance

of H_a) when it is true. If H_o is not rejected, we can conclude that

the difference between the observed and theoretical distributions is due to sampling error, since we did not observe the population.

The test procedure is best illustrated by application to the FMS case distribution, at an α of 0.05:

1. H_o: The population distribution of arriving cases is Poisson distributed.

2. H_a: The population distribution of arriving cases is not Poisson distributed.

3. The number of degrees of freedom is the number of frequency classes minus the number of requirements of the Poisson distribution (λ and total frequency = 2). In order to meet the condition of five or more observations per class, the data of Table 2 must be reorganized as shown in Table A.1. The modified number of frequency classes is eight, resulting in d.f. = 8-2 = 6. The table value for $\chi^2_{\alpha = 0.05}$ (6 d.f.) = 12.592.

4. Criterion: Reject H_o (accept H_a) if $\chi^2 > 12.592$; do not reject H_o if $\chi^2 \leq 12.592$, when χ^2 is computed using the previously-defined formula.

5. Since χ^2 (2.087) $< \chi^2$ (12.592), H_o is not rejected; the observed frequency distribution is Poisson.

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TABLE A-1.   FREQUENCY DISTRIBUTION-FMS CASE ARRIVALS

No. of Cases Arriving Per Day (X)	f_o	f_e	(f_o-f_e)	$(f_o-f_e)^2$	$\dfrac{(f_o-f_e)^2}{f_e}$
1 or fewer	5	3.48	1.52	2.3104	0.664
2	7	6.92	0.08	0.0064	0.001
3	10	11.07	-1.07	1.1449	0.103
4	15	13.30	1.70	2.8900	0.217
5	10	12.75	-2.75	7.5625	0.593
6	10	10.21	-0.21	0.0441	0.004
7	6	7.00	-1.00	1.0000	0.143
8 or more	10	8.27	1.73	2.9929	0.362
	73	73.00	0	$\chi^2 =$	2.087

APPENDIX B

QUEUEING MODEL SOLUTIONS

QUEUEING MODEL - MULTIPLE STATION

```
        k =        4.000
   lambda =        5.300
       mu =        1.460
      rho =         .907

       P° =         .010262
     Pn,k =         .803047
     E(w) =        1.487124
     E(m) =        7.881761
     E(n) =       11.511898
     E(t) =        2.172056
```

QUEUEING MODEL - MULTIPLE STATION

```
        k =        5.000
   lambda =        5.300
       mu =        1.460
      rho =         .726

       P° =         .021928
     Pn,k =         .420474
     E(w) =         .210237
     E(m) =        1.114257
     E(n) =        4.744394
     E(t) =         .895168
```

QUEUEING MODEL - MULTIPLE STATION

```
        k =        6.000
   lambda =        5.300
       mu =        1.460
      rho =         .605

       P° =         .025171
     Pn,k =         .202552
     E(w) =         .058541
     E(m) =         .310267
     E(n) =        3.940404
     E(t) =         .743472
```

QUEUEING MODEL - MULTIPLE STATION

```
        k =        7.000
   lambda =        5.300
       mu =        1.460
      rho =         .518

       P° =         .026121
     Pn,k =         .089434
     E(w) =         .018177
     E(m) =         .096342
     E(n) =        3.726479
     E(t) =         .703109
```

QUEUEING MODEL - MULTIPLE STATION

```
        k =          8.000
   lambda =          5.300
       mu =          1.460
      rho =           .453

       P° =           .026401
     Pn,k =           .036150
     E(w) =           .005666
     E(m) =           .030030
     E(n) =          3.660167
     E(t) =           .690597
```

QUEUEING MODEL - MULTIPLE STATION

```
        k =          9.000
   lambda =          5.300
       mu =          1.460
      rho =           .403

       P° =           .026482
     Pn,k =           .013389
     E(w) =           .001707
     E(m) =           .009051
     E(n) =          3.639188
     E(t) =           .686639
```

QUEUEING MODEL - MULTIPLE STATION

```
        k =         10.000
   lambda =          5.300
       mu =          1.460
      rho =           .363

       P° =           .026504
     Pn,k =           .004356
     E(w) =           .000489
     E(m) =           .002596
     E(n) =          3.632733
     E(t) =           .685421
```

QUEUEING MODEL - MULTIPLE STATION

```
        k =         11.000
   lambda =          5.300
       mu =          1.460
      rho =           .330

       P° =           .026510
     Pn,k =           .001430
     E(w) =           .000132
     E(m) =           .000704
     E(n) =          3.630841
     E(t) =           .685064
```

```
        k  =           12.000
   lambda  =            5.300
       mu  =            1.460
      rho  =             .302

       P°  =             .026512
     Pn,k  =             .000415
     E(w)  =             .000034
     E(m)  =             .000180
     E(n)  =            3.630317
     E(t)  =             .684965
```

QUEUEING MODEL - MULTIPLE STATION

```
        k  =           13.000
   lambda  =            5.300
       mu  =            1.460
      rho  =             .279

       P°  =             .026512
     Pn,k  =             .000112
     E(w)  =             .000008
     E(m)  =             .000043
     E(n)  =            3.630180
     E(t)  =             .684939
```

QUEUEING MODEL - MULTIPLE STATION

```
        k  =           14.000
   lambda  =            5.300
       mu  =            1.460
      rho  =             .259

       P°  =             .026512
     Pn,k  =             .000028
     E(w)  =             .000001
     E(m)  =             .000009
     E(n)  =            3.630146
     E(t)  =             .684933
```

QUEUEING MODEL - MULTIPLE STATION

```
        k  =           15.000
   lambda  =            5.300
       mu  =            1.460
      rho  =             .242

       P°  =             .026512
     Pn,k  =             .000006
     E(w)  =             .000000
     E(m)  =             .000002
     E(n)  =            3.630139
     E(t)  =             .684931
```

```
      k =              4.000
 lambda =              5.300
     mu =              2.000
    rho =               .662

     P° =               .061157
   Pn,k =               .372348
   E(w) =               .137906
   E(m) =               .730905
   E(n) =              3.380905
   E(t) =               .637906
```

QUEUEING MODEL - MULTIPLE STATION

$$k = 5.000$$
$$lambda = 4.780$$
$$mu = 1.150$$
$$rho = .831$$

$$P^\circ = .010051$$
$$Pn,k = .016027$$
$$E(w) = .635079$$
$$E(m) = 3.035682$$
$$E(n) = 7.192204$$
$$E(t) = 1.504645$$

QUEUEING MODEL - MULTIPLE STATION

$$k = 6.000$$
$$lambda = 4.780$$
$$mu = 1.150$$
$$rho = .692$$

$$P^\circ = .013917$$
$$Pn,k = .324440$$
$$E(w) = .153037$$
$$E(m) = .731520$$
$$E(n) = 4.888042$$
$$E(t) = 1.022603$$

QUEUEING MODEL - MULTIPLE STATION

$$k = 7.000$$
$$lambda = 4.780$$
$$mu = 1.150$$
$$rho = .593$$

$$P^\circ = .015112$$
$$Pn,k = .158220$$
$$E(w) = .048385$$
$$E(m) = .231282$$
$$E(n) = 4.387804$$
$$E(t) = .917950$$

QUEUEING MODEL - MULTIPLE STATION

$$k = 8.000$$
$$lambda = 4.780$$
$$mu = 1.150$$
$$rho = .519$$

$$P^\circ = .015491$$
$$Pn,k = .071247$$
$$E(w) = .016119$$
$$E(m) = .077050$$
$$E(n) = 4.233571$$
$$E(t) = .885684$$

QUEUEING MODEL - MULTIPLE STATION

```
         k =      9.000
    lambda =      4.780
        mu =      1.150
       rho =       .461

        P° =       .015610
      Pn,k =       .029601
      E(w) =       .005314
      E(m) =       .025402
      E(n) =      4.181924
      E(t) =       .874879
```

QUEUEING MODEL - MULTIPLE STATION

```
         k =     10.000
    lambda =      4.780
        mu =      1.150
       rho =       .415

        P° =       .015647
      Pn,k =       .011357
      E(w) =       .001690
      E(m) =       .008078
      E(n) =      4.164600
      E(t) =       .871255
```

QUEUEING MODEL - MULTIPLE STATION

```
         k =     11.000
    lambda =      4.780
        mu =      1.150
       rho =       .377

        P° =       .015657
      Pn,k =       .004033
      E(w) =       .000512
      E(m) =       .002450
      E(n) =      4.158971
      E(t) =       .870077
```

QUEUEING MODEL - MULTIPLE STATION

```
         k =     12.000
    lambda =      4.780
        mu =      1.150
       rho =       .346

        P° =       .015660
      Pn,k =       .001330
      E(w) =       .000147
      E(m) =       .000704
      E(n) =      4.157226
      E(t) =       .869712
```

QUEUEING MODEL - MULTIPLE STATION

```
        k  =           13.000
    lambda  =            4.780
        mu  =            1.150
       rho  =             .319

        P°  =             .015661
      Pn,k  =             .000408
      E(w)  =             .000040
      E(m)  =             .000192
      E(n)  =            4.156713
      E(t)  =             .869605
```

QUEUEING MODEL - MULTIPLE STATION

```
        k  =           14.000
    lambda  =            4.780
        mu  =            1.150
       rho  =             .296

        P°  =             .015661
      Pn,k  =             .000117
      E(w)  =             .000010
      E(m)  =             .000049
      E(n)  =            4.156571
      E(t)  =             .869575
```

QUEUEING MODEL - MULTIPLE STATION

```
        k  =           15.000
    lambda  =            4.780
        mu  =            1.150
       rho  =             .277

        P°  =             .015661
      Pn,k  =             .000031
      E(w)  =             .000002
      E(m)  =             .000012
      E(n)  =            4.156533
      E(t)  =             .869567
```

QUEUEING MODEL - MULTIPLE STATION

```
        k  =           16.000
    lambda  =            4.780
        mu  =            1.150
       rho  =             .259

        P°  =             .015661
      Pn,k  =             .000008
      E(w)  =             .000000
      E(m)  =             .000002
      E(n)  =            4.156524
      E(t)  =             .869565
```

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13. ABSTRACT

The application of queueing, or waiting-line, theory to the the review and validation of Foreign Military Sales (FMS) cases by the Cost Analysis office of an Army Materiel Development and Readiness Command major subordinate command is presented in this report. Queueing analysis is applied to determine the behavior characteristics of the processing system so that management can take appropriate action to reduce the total time in the system. The primary characteristics analyzed are the expected length of the queue, the expected number of cases in the system, the expected waiting time, and the expected turnaround time. Queueing theory is described, and the application to FMS cases presented, to also serve as an aid in the solution of similar types of waiting-line problems.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Queueing Model						
Waiting-Line Model						
Operations Research Model						
Management						
Management Technique						
Operations Engineering						